IV. Conclusions

A compliance matrix is introduced to describe the quasi-static characteristics of unsymmetrical broadside-coupled striplines of unequal width, and the upper-bound variational expressions for the matrix elements are derived. These formulas show the transformation of the anisotropic structure into the equivalent isotropic structure, and it requires the shift of the strips in addition to the replacement of the substrate. Numerical results obtained by accurate numerical procedure show that the line characteristics are influenced by the configuration of the strip conductors significantly.

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Calculation of the Interaction Between the Fringing Capacitance of Symmetrical Stripline Using the Finite Element Method

J.R. NORTIER

Abstract —**This paper calculates the interaction between the fringing capacitance of symmetrical stripline using the finite-element method. The fringing capacitance interaction is presented as a function of T/B and W/B. This enables the characteristic impedance of narrow, high-impedance lines to be accurately and simply calculated.**

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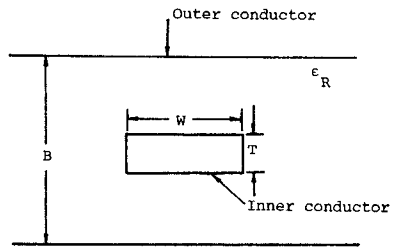

Fig. 1. Cross section of the symmetrical stripline

I. Introduction

Consider the cross section of the symmetrical stripline as shown in Fig. 1. Using a notation similar to that of Getsinger [1], the total capacitance per unit length C_0 of the structure is given by

$$C_0 = 2C_p + 4C_f(W/B, T/B) \tag{1}$$

and

$$C_p = \frac{2\epsilon W}{B-T}. \tag{2}$$

The permittivity of the dielectric medium between the conductors is $\epsilon = \epsilon_0 \epsilon_R$, where ϵ_0 is the permittivity of free space and ϵ_R is the relative permittivity of the dielectric medium.

If W is large enough, $C_f(W/B, T/B)$ tends to $C_f(\infty, T/B)$, and it can be said that there is no interaction between the fringing capacitances of the line. The interaction between the fringing capacitances can be quantified as

$$\Delta C_f = C_f(\infty, T/B) - C_f(W/B, T/B). \tag{3}$$

The total capacitance per unit length of the line is given by

$$C_0 = 2C_p + 4C_f(\infty, T/B) + 4\Delta C_f. \tag{4}$$

The total capacitance per unit length can be approximated by

$$C_0' \approx 2C_p + 4C_f(\infty, T/B) \tag{5}$$

where from [2]

$$C_f(\infty, T/B) = \frac{\epsilon}{\pi}\{2\xi \ln(\xi+1) - (\xi-1)\ln(\xi^2-1)\} \tag{6}$$

and

$$\xi = 1/(1-T/B). \tag{7}$$

The approximation will, however, deteriorate as W decreases and fringing capacitance interaction occurs.

Getsinger [1] uses a correction factor when computing C_0' for narrow lines, but this is based on the case for $T/B = 0$ as calculated by Cohn [2], and is not necessarily accurate for other T/B values. Riblet [3]–[5] has investigated the case of the odd-mode fringing capacitance interaction ΔC_{f0} where the side-wall influence is included. He has calculated the upper and lower limits of ΔC_{f0} from which an approximation to ΔC_{f0} can be made. As side walls are considered, however, these results cannot be directly applied to the geometry of Fig. 1.

This paper provides an accurate value of ΔC_f as a function of T/B and W/B for the case of the symmetrical stripline. This enables the accurate calculation of C_0 in a simple manner, and is especially useful for narrow, high-impedance lines.

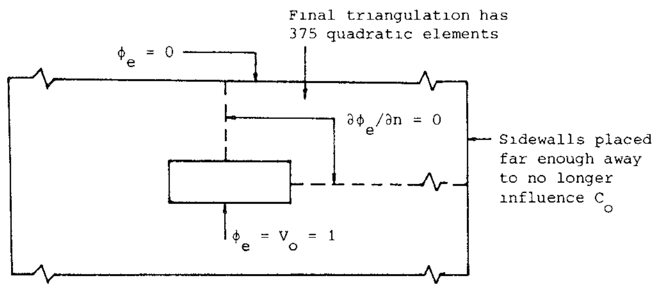

Fig. 2. Boundary conditions for the calculation of C_{0u}. Due to symmetry, only a quarter of the structure is considered.

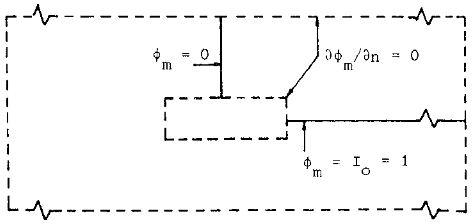

Fig. 3. Boundary conditions for the calculation of C_{0l}.

II. Calculation of ΔC_f Using the Finite Element Method

The finite-element method is used to calculate the total capacitance per unit length of the stripline geometry [6]. A TEM mode of propagation is assumed. Thus, if $\phi(x, y)$ is a scalar potential, the electric and magnetic fields can be found from the solution of the scalar, two-dimensional Laplace's equation, $\nabla^2\phi(x, y) = 0$, subject to the appropriate boundary conditions. To model the geometry of Fig. 1, the sidewalls are placed sufficiently far away so that they no longer have any influence on C_0. In using the finite-element method, the capacitance per unit length is simply related to the variational expression for the stored energy which is to be minimized. The accuracy of the method can, however, be improved by calculating both upper C_{0u} and lower C_{0l} bounds for the capacitance per unit length [7]–[9].

The upper bound is given by

$$C_{0u} = \frac{\epsilon_0\epsilon_R}{V_0^2} \iint_s |\nabla\phi_e(x, y)|^2 \, dx\, dy \tag{8}$$

where V_0 is a unit potential and $-\nabla\phi_e(x, y)$ equals $\overline{E}$, the electric field. The natural boundary conditions as shown in Fig. 2 are specified.

The lower bound is given by

$$1/C_{0l} = \frac{1}{\epsilon_0\epsilon_R I_0^2} \iint_s |\nabla\phi_m(x, y)|^2 \, dx\, dy \tag{9}$$

where I_0 is a unit current and $-\nabla\phi_m(x, y)$ equals $\overline{H}$, the magnetic field. The boundary conditions are interchanged as shown in Fig. 3.

The finite-element method is used to solve C_{0u} and C_{0l} as a function of T/B and W/B from (8) and (9) with the appropriate boundary conditions of Figs. 2 and 3. Due to symmetry, only a quarter of the structure need be considered.

For the case $T/B = 0.4$ and $W/B = 0.3$, the convergence of C_{0l} and C_{0u} is shown in Table I with $C_0 = 1/2(C_{0u} + C_{0l})$. Sufficient accuracy is obtained for the calculation of ΔC_f by using 375 quadratic elements in a quarter of the structure.

TABLE I
Calculation of C_0/ϵ From its Upper and Lower Bounds, Using the Code Discussed in [6], for the Case $T/B = 0.4$ and $W/B = 0.3$ with $\epsilon_R = 1$.

NUMBER OF QUADRATIC ELEMENTS	C_{ou}/ϵ	$C_{o\ell}/\epsilon$	C_o/ϵ
40	5.970080	5.478499	5.72429
150	5.712680	5.635358	5.674019
375	5.687800	5.650140	5.668970

The value of ΔC_f can be calculated from (4) and is shown in Fig. 4 for various values of T/B and W/B.

For the case $W/B = 0.35$ and $T/B = 0$, Cohn [2] states that the error in total capacitance due to interaction of the fringing fields is approximately 1.24 percent. From Fig. 4, (4), and (5), the error in total capacitance $(C_0' - C_0)/C_0'$ is calculated as 1.16 percent, thus being slightly less than Cohn's value.

As shown in Fig. 4, the interaction of fringing capacitance decreases as T/B increases. The value of W/B for a fixed T/B where ΔC_f becomes negligible, and the approximation C_0' could be used, can also be obtained from Fig. 4. For example, for the case $W/B = 0.21$ and $T/B = 0.4$, we have $(W/B)/(1 - T/B) = 0.35$, and the error in the total capacitance due to interaction is only 0.22 percent. Thus, the correction as suggested by Getsinger [1] based on Cohn's calculation for the $T/B = 0$ case with a 1.24-percent error is somewhat too pessimistic and will result in an over-correction for the line width.

III. Application of ΔC_f in Calculating Z_0

Once the total capacitance per unit length C_0 is known from (4), the characteristic impedance Z_0 can be calculated from

$$Z_0\sqrt{\epsilon_R} = \frac{\sqrt{u_0/\epsilon_0}}{C_0/\epsilon}. \tag{10}$$

If ΔC_f is unknown or can be ignored, C_0' from (5) can be used to calculate Z_0' in similar fashion.

In Table II, Z_0 and Z_0' have been calculated for various values of W/B and T/B. For narrow lines with small T/B values, the importance of considering ΔC_f is clearly demonstrated. The results of Z_0 calculated from (4) also compare favorably with the exact values obtained from [10].

IV. Conclusions

The interaction between the fringing capacitance of the symmetrical stripline has been calculated using the finite-element method. It is shown that the interaction of fringing capacitance decreases as T/B increases for a fixed W/B and thus Getsinger's [1] correction factor for narrow lines will lead to overcorrection for $T/B > 0$. A simple, easy to use method for calculating the characteristic impedance and capacitance per unit length of narrow high-impedance lines using (4) and Fig. 4 has been presented. This eliminates the need for any correction factor for narrow lines, and the results compare favorably with published data as shown in Table II.

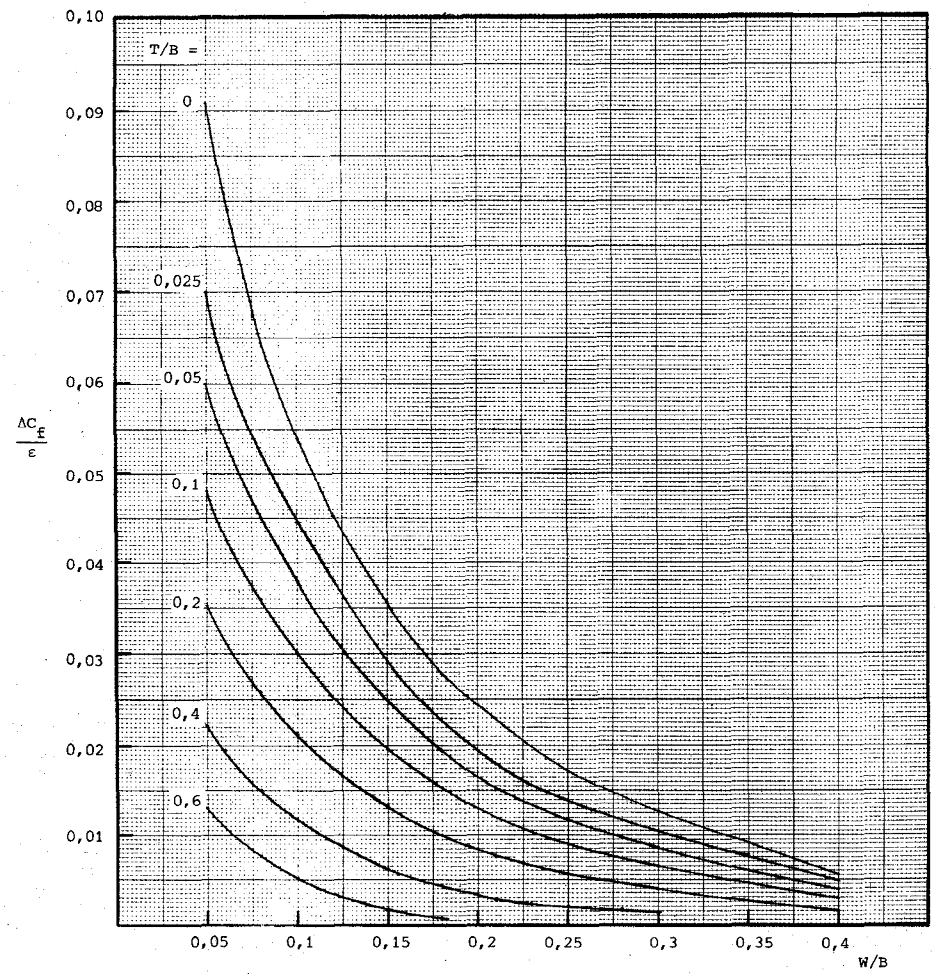

Fig. 4. The interaction of fringing capacitance $\Delta C_f/\epsilon$ as a function of W/B and T/B for the symmetrical stripline.

TABLE II
CALCULATED VALUES $Z_0\sqrt{\epsilon_R}$ AND $Z_0'\sqrt{\epsilon_R}$ FOR VARIOUS VALUES OF T/B AND W/B

T/B	W/B	$Z_o\sqrt{\epsilon_R}$ From Eqn.(4) (ohms)	$Z'_o\sqrt{\epsilon_R}$ From Eqn.(5) (ohms)	ERROR $(Z_o-Z'_o)\sqrt{\epsilon_R}$ (ohms)	$Z_o\sqrt{\epsilon_R}$ EXACT From [10] (ohms)
0.0	0.05	235.22	191.71	43.51	235.66
0.0	0.21	149.91	144.61	5.30	150.13
0.0	0.4	113.17	111.95	1.22	112.83
0.05	0.21	129.91	127.23	2.68	129.98
0.05	0.4	100.59	100.17	0.42	100.63
0.2	0.05	131.12	124.98	6.14	131.14
0.2	0.21	99.55	98.76	0.79	99.62
0.2	0.4	79.18	79.07	0.11	79.19
0.4	0.05	96.13	94.00	2.13	96.15
0.4	0.21	74.40	74.24	0.16	74.45
0.6	0.05	71.68	70.97	0.71	71.72
0.6	0.3	48.26	48.25	0.01	48.26

Accurate knowledge of the interaction of the fringing capacitance could also be applied to the calculation of the end-effect compensation and resonant lengths of narrow lines.

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